

# On the kinetic theory of turbulence in plasmas

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From the nonlinear Vlasov equation, a turbulence scattering term is found to describe the stochasticity on the time scale longer than the turbulence correlation time. The evolution of the plasma distribution is determined by the well-understood unperturbed motion of charged particles, with the effects of fluctuating part of fields described by the turbulence scattering term, from which, one can identify various important physics, covering from the linear and quasilinear to the nonlinear regime, in particular the collections between the widely used Kadomtsev-Pogutse's equation and Frieman-Chen's equation.

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Plasma transport is one of the most challenging scientific problems. It is theoretically treated by solving the kinetic equation that determines the evolution of the particle distribution in the phase space, and the kinetic equation is nonlinearly coupled with Maxwell's equations, because the motion of charged particles depends on the electric magnetic fields and meanwhile the fields depend on the distribution of the charged particles [1–7]. Generally, the fields can be decomposed into the fluctuating part and the averaged part [1], and the motion of charged particles in averaged fields is usually well-understood. The fluctuation of fields can be due to the particle discreteness and the collective instabilities of the plasma, which are known as collisional process [2–4] and micro-turbulence [5–7], respectively. Macroscopically, both the collisional transport [2–4] and the turbulent transport [6–9] can be described by diffusion-convection fluxes.

Up to the present, the collisional transport and the turbulent transport are theoretically investigated in different ways. In the well-established collisional transport theory [2, 4], the particle motion due to the fields fluctuation induced by the particle discreteness is decoupled from the unperturbed motion in the averaged fields [1], and the decoupled motion is described by the Landau collision operator [3], which can be written in a Fokker-Planck form. In the turbulent transport theory and numerical simulation, however, the particle motion due to the field fluctuation is usually not decoupled, and the particle trajectory has to be evaluated with the full fields [6, 7]. In this way, one has to solve the nonlinear Vlasov equation or its gyro-averaged version for the magnetic confinement fusion plasma, by following the particle trajectory in the full fields [10–13] [14–18].

Recently, it has been proposed that the particle motion in the fluctuation fields due to the collective process can be decoupled from the unperturbed motion, by using the Hamiltonian Lie-transform perturbation method, and the quasilinear transport theory has been successfully demonstrated [9]. In this paper, by using this new method, we will show that, in a nonlinear turbulent plasma, the particle motion due to the fluctuating part of fields can be decoupled from the well-understood unperturbed motion, and the effects of fluctuating fields on the particle distribution can be put into a turbulence scattering term, which is written in a Fokker-Planck form as a divergence of diffusive-convective flux in the phase space. The turbulent transport and the collisional transport can be theoretically treated in a unified way.

We begin with the general kinetic theory of plasma transport [1].

$$\mathcal{V}(f_s) = 0, \quad (1a)$$

$$\mathcal{V} \equiv \partial_t + \mathbf{v} \cdot \partial_{\mathbf{x}} + \frac{e_s}{m_s} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \partial_{\mathbf{v}}, \quad (1b)$$

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \left( \rho_{ex} + \Sigma_s \int d^3 \mathbf{v} e_s f_s \right), \quad (2a)$$

$$\nabla \times \mathbf{B} = \mu_0 \left( \mathbf{j}_{ex} + \Sigma_s \int d^3 \mathbf{v} e_s \mathbf{v} f_s \right) + \frac{1}{c^2} \partial_t \mathbf{E}, \quad (2b)$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \quad (2c)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (2d)$$

where  $f_s(\mathbf{x}, \mathbf{v}, t)$  is the distribution function of particle species  $s$  at time  $t$ , with  $\mathbf{x}$  and  $\mathbf{v}$  the particle position and velocity, respectively;  $e_s$  and  $m_s$  the charge and mass of the particle, respectively;  $\mathbf{E}$  and  $\mathbf{B}$  the electric and magnetic fields, respectively.  $\rho_{ex}$  and  $\mathbf{j}_{ex}$  are electrical charge density and current density due to the external sources, respectively. In the following, the subscript  $s$  shall be omitted for simplicity.

For comparison, we write down the kinetic equation for collisional transport [3].

$$\mathcal{V}_0(f) = \mathcal{C}(f) \equiv -\partial_{\mathbf{v}} \cdot \left[ \left( \mathbf{a}_c - \frac{1}{2} \mathbf{d}_c \cdot \partial_{\mathbf{v}} \right) f \right], \quad (3a)$$

$$\mathcal{V}_0 \equiv \partial_t + \mathbf{v} \cdot \partial_{\mathbf{x}} + \frac{e}{m} (\mathbf{E}_0 + \mathbf{v} \times \mathbf{B}_0) \cdot \partial_{\mathbf{v}}, \quad (3b)$$

where the velocity space convection vector  $\mathbf{a}_c$  and diffusion tensor  $\mathbf{d}_c$  depend on [3]  $f$ .  $\mathbf{E}_0$  and  $\mathbf{B}_0$  are averaged fields, with the fluctuation due to particle discreteness removed to the collisional scattering term,  $\mathcal{C}(f)$ . Usually, the effects of the micro-instabilities are not included in collisional transport theory.

In the following, to concentrate on the turbulent transport, we shall deal with Eq. (1a), with  $\mathbf{E} = \mathbf{E}_0 + \delta \mathbf{E}$  and  $\mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B}$  understood as the fields without fluctuation due to the particle discreteness, and  $\mathbf{E}_0$  and  $\mathbf{B}_0$  understood as the equilibrium fields that do not include the fluctuations  $(\delta \mathbf{E}, \delta \mathbf{B})$  due to micro-instabilities.

We begin with the particle motion in the unperturbed electromagnetic fields. The unperturbed fundamental one-form (phase space Lagrangian) [19] is written in terms of the noncanonical variables  $\mathbf{z} = (\mathbf{x}, \mathbf{v})$  as

$$\gamma_0 = [m\mathbf{v} + e\mathbf{A}_0(\mathbf{x})] \cdot d\mathbf{x} - H_0(\mathbf{x}, \mathbf{v}) dt, \quad (4)$$

with the unperturbed Hamiltonian  $H_0 = (1/2)m|\mathbf{v}|^2 + e\Phi_0(\mathbf{x})$ .  $\mathbf{E}_0 = -\nabla\Phi_0$ .  $\mathbf{B}_0 = \nabla \times \mathbf{A}_0$ .

The well-understood equations of unperturbed motion are

$$\dot{z}_0^i = \{z^i, H_0\} = J_0^{ij} \partial_j H_0, \quad (5)$$

with  $J_0^{ij} = \{z^i, z^j\}$ , and the unperturbed Poisson bracket [19]

$$\{f, g\} = \frac{1}{m} (\partial_{\mathbf{x}} f \cdot \partial_{\mathbf{v}} g - \partial_{\mathbf{v}} f \cdot \partial_{\mathbf{x}} g) - \frac{e}{m^2} \mathbf{B}_0 \cdot \partial_{\mathbf{v}} f \times \partial_{\mathbf{v}} g. \quad (6)$$

Introduce the perturbations of the electric field  $\delta\mathbf{E} = -\nabla\delta\Phi(\mathbf{x}, t) - \partial_t\delta\mathbf{A}(\mathbf{x}, t)$  and the magnetic field  $\delta\mathbf{B} = \nabla \times \delta\mathbf{A}$ , which are due to the micro-instabilities. The fundamental one-form is separated into the unperturbed part and the perturbation part,  $\gamma = \gamma_0 + \gamma_1$ , with

$$\gamma_1 = e\delta\mathbf{A}(\mathbf{x}, t) \cdot d\mathbf{x} - e\delta\Phi(\mathbf{x}, t)dt. \quad (7)$$

We shall endeavor to decouple the perturbed motion due to the turbulence from the unperturbed motion by using the recently proposed method of I-transform [9], which, as a special Lie-transform [20, 21], makes the equations of motion in the perturbed fields formally identical to the unperturbed motion. The ordering parameter is  $\epsilon_\delta \sim |\frac{\delta\mathbf{E}}{\mathbf{B}_0 v_{th}}| \sim |\frac{\delta\mathbf{B}}{\mathbf{B}_0}| \ll 1$ , with  $v_{th}$  the thermal speed of the particle. To  $\mathcal{O}(\epsilon_\delta^2)$ , the phase-space Lie transform [20, 21] from the old variables  $\mathbf{z}$  to the new variables  $\bar{\mathbf{z}}$  is

$$\bar{z}^i = z^i + G_1^i(\mathbf{z}) + G_2^i + \frac{1}{2} G_1^j \partial_j G_1^i, \quad (8a)$$

$$z^i = \bar{z}^i - G_1^i(\bar{\mathbf{z}}) - G_2^i + \frac{1}{2} G_1^j \partial_j G_1^i, \quad (8b)$$

where  $\mathbf{G}_1$  and  $\mathbf{G}_2$  are the first order and the second order Lie-transform generating vectors, respectively.

The distribution function (a scalar) is transformed according to  $\bar{f}(\bar{\mathbf{z}}) = f(\mathbf{z})$ , which can be written up to  $\mathcal{O}(\epsilon_\delta^2)$  as

$$\bar{f} = f - (G_1^i + G_2^i) \partial_i f + \frac{1}{2} G_1^i \partial_i G_1^j \partial_j f, \quad (9a)$$

$$f = \bar{f} + (G_1^i + G_2^i) \partial_i \bar{f} + \frac{1}{2} G_1^i \partial_i G_1^j \partial_j \bar{f}. \quad (9b)$$

The straightforward calculation of the I-transform begins with  $\gamma_0$  and  $\gamma_1$ , and it is similar to Ref. 9. The results needed are summarized as follows.

The gauge functions of the I-transform,  $S_n$ , are solved by

$$\mathcal{V}_0(S_n) = e(\delta\Psi_n - \mathbf{v} \cdot \delta\mathcal{A}_n). \quad (10)$$

For the low-frequency turbulence, the gyro-phase dependent part of  $S_n$  can be analytically solved [9] to decouple the fast gyro-motion.

The I-transform generating vector can be written as

$$G_n^x = -\frac{1}{m}\partial_v S_n, \quad (11a)$$

$$G_n^v = \frac{1}{m}\partial_x S_n + \frac{e}{m^2}\mathbf{B}_0 \times \partial_v S_n + \frac{e}{m}\delta\mathcal{A}_n. \quad (11b)$$

The effective potentials are given by

$$[\delta\mathcal{A}_1, \delta\Psi_1] = [\delta\mathbf{A}, \delta\Phi], \quad (12a)$$

$$[\delta\mathcal{A}_2, \delta\Psi_2] = [\frac{1}{2}G_1^x \times \delta\mathbf{B}, \frac{1}{2}G_1^x \cdot \delta\mathbf{E}]. \quad (12b)$$

With the I-transform generating vectors given above, the transformed fundamental one-form is identical to the unperturbed one,

$$\bar{\gamma}_0 = [m\bar{\mathbf{v}} + e\mathbf{A}_0(\bar{\mathbf{x}})] \cdot d\bar{\mathbf{x}} - H_0(\bar{\mathbf{x}}, \bar{\mathbf{v}}) dt, \quad (13)$$

and hence the equations of motion are simply

$$d\bar{z}^i/dt = \{\bar{z}^i, H_0\} = J_0^{ij}\partial_j H_0. \quad (14)$$

The Vlasov equation is transformed to

$$\mathcal{V}_0(\bar{f}) = 0. \quad (15)$$

It is not hard to show that  $\mathbf{G}_n$ 's are incompressible flows in the phase space [9],

$$\partial_{\mathbf{z}} \cdot \mathbf{G}_n = 0. \quad (16)$$

To proceed, we make some remarks on the I-transform.  $\mathbf{G}_n$ 's describe the excursion from the unperturbed orbit. In describing the turbulence wave scattering, Eq. (10) may be understood as the stochastic equations [6]. It is important to understand that the I-transform is a perturbation theory; its validity depends on the condition that the real orbit does not deviate much from the unperturbed orbit. When the perturbation is weak, this

condition is well satisfied; clearly, this is the case of linear and quasi-linear theory discussed in Ref. 9. However, in the nonlinear stage, the perturbation of fields can be strong, and hence the real orbit of the particle may deviate significantly away from the unperturbed orbit in a long-time. Therefore, in the nonlinear stage, one can only apply the I-transform in a small time interval.

To evolve the particle distribution within a small time interval  $\Delta t$ , we denote the distribution function at time  $t - \Delta t$  as  $f(\mathbf{z}, t - \Delta t)$ . To apply the I-transform to find the distribution function at time  $t$ , one should keep in mind that it is the functions rather than the values that the Lie-transform operates on [20, 21].

Set  $S_n(\mathbf{z}, t - \Delta t) = 0$ , we have  $\mathbf{G}_n(\mathbf{z}, t - \Delta t) = 0$ . This means that we do not make any transform at time  $t - \Delta t$ ; therefore, according to Eq. (9a), we have

$$\bar{f}(\bar{\mathbf{z}}, t - \Delta t) = f(\bar{\mathbf{z}}, t - \Delta t). \quad (17)$$

Integrating Eq. (10) along the unperturbed orbit from  $t - \Delta t$  to  $t$ , we can find  $S_n(\mathbf{z}, t)$  and  $\mathbf{G}_n(\mathbf{z}, t)$ . Since this is an infinitesimal I-transform, we denote the infinitesimal gauge functions and the infinitesimal generating vectors as  $s_n(\mathbf{z}, t)$  and  $\mathbf{g}_n(\mathbf{z}, t)$ , respectively. Integrating Eq. (15), we found

$$\bar{f}(\bar{\mathbf{z}}, t) = \bar{f}[\bar{\mathbf{z}} - \Delta\mathbf{z}(\bar{\mathbf{z}}), t - \Delta t], \quad (18a)$$

$$-\Delta\mathbf{z}(\mathbf{z}) = \int_t^{t-\Delta t} dt \{ \mathbf{z}, H_0 \}. \quad (18b)$$

Substituting Eq. (17), we found

$$\bar{f}(\bar{\mathbf{z}}, t) = f[\bar{\mathbf{z}} - \Delta\mathbf{z}(\bar{\mathbf{z}}), t - \Delta t]. \quad (19)$$

Using Eq. (9b) and Eq. (16), we found

$$f(\mathbf{z}, t) = \bar{f}(\mathbf{z}, t) + \partial_{\mathbf{z}} \cdot \left[ (\mathbf{g}_1 + \mathbf{g}_2) \bar{f} + \frac{1}{2} \mathbf{g}_1 \mathbf{g}_1 \cdot \partial_{\mathbf{z}} \bar{f} \right]. \quad (20)$$

Collecting Eq. (19) and Eq. (20), we found

$$\mathcal{V}_0(f) = -\partial_{\mathbf{z}} \cdot \left\{ \left[ \frac{-(\mathbf{g}_1 + \mathbf{g}_2)}{\Delta t} - \frac{1}{2} \frac{\mathbf{g}_1 \mathbf{g}_1}{\Delta t} \cdot \partial_{\mathbf{z}} \right] f \right\}. \quad (21)$$

By introducing the nonlinear turbulence scattering term  $\mathcal{T}(f)$ , the kinetic equation can be written as

$$\mathcal{V}_0(f) = \mathcal{T}(f) \equiv -\partial_{\mathbf{z}} \cdot \left[ \left( \mathbf{a}_{\mathcal{T}} - \frac{1}{2} \mathbf{d}_{\mathcal{T}} \cdot \partial_{\mathbf{z}} \right) f \right]. \quad (22)$$

The phase space convection vector  $\mathbf{a}_{\mathcal{T}}$  and the symmetric diffusion tensor  $\mathbf{d}_{\mathcal{T}}$  are given by

$$a_{\mathcal{T}}^i = a_{\mathcal{T},1}^i + a_{\mathcal{T},2}^i \equiv \frac{-g_1^i}{\Delta t} + \frac{-g_2^i}{\Delta t}, \quad (23)$$

$$d_{\mathcal{T}}^{ij} = \frac{g_1^i g_1^j}{\Delta t}. \quad (24)$$

The finiteness of  $\mathbf{d}_{\mathcal{T}}$  and  $\mathbf{a}_{\mathcal{T},2}$  in  $\mathcal{T}(f)$  is due to the stochasticity developed by the nonlinearity on the time scale  $\Delta t$ ; similarly, the finiteness of  $\mathbf{d}_{\mathcal{C}}$  in  $\mathcal{C}(f)$  is due to the randomness of the collisional scattering events on the time scale longer than the typical correlation time in collisional process [1]. At this point, the scale of  $\Delta t$  should be further clarified in connection with the nonlinear stochasticity [6]. Let  $\delta t$  be the turbulence correlation time. Note the fact that the stochasticity displays on a time scale longer than  $\delta t$ , while in a time scale shorter than  $\delta t$ , the stochasticity does not display itself. If one looks on a time scale  $\Delta t > \delta t$ , the randomness due to the stochasticity makes  $\mathbf{d}_{\mathcal{T}}$  and  $\mathbf{a}_{\mathcal{T},2}$  finite; if one looks on the time scale  $\Delta t \ll \delta t$ ,  $\mathbf{d}_{\mathcal{T}}$  and  $\mathbf{a}_{\mathcal{T},2}$  are indeed unimportant. Clearly,  $\mathbf{d}_{\mathcal{T}}$  and  $\mathbf{a}_{\mathcal{T},2}$  describe the turbulent dissipation on the time scale longer than  $\delta t$ . In fact, if one determines to resolve the time scale much shorter than  $\delta t$ ,  $\mathbf{d}_{\mathcal{T}}$  and  $\mathbf{a}_{\mathcal{T},2}$  can be ignored, and the  $\mathbf{a}_{\mathcal{T},1} \cdot \partial_z f$  term retained in Eq. (22) describes all the nonlinearity; this is essentially the underlying physics behind Frieman-Chen's equation [10] in gyro-averaged form; note that on this time scale,  $\mathbf{a}_{\mathcal{T},1}$  is the first order velocity in phase space. The robustness of the turbulence scattering term shall be demonstrated in the following applications.

Note that the temporal accumulation of  $\mathbf{g}_1$  (the infinitesimal I-transform) is  $\mathbf{G}_1$  (the long-time I-transform). It is not hard to show that the present theory recovers the linear theory and quasilinear theory. To do this we separate the distribution function into the fluctuation part  $\tilde{f}$  and the ensemble averaged part  $\langle f \rangle$ , that is  $f = \langle f \rangle + \tilde{f}$ ,  $\langle \tilde{f} \rangle = 0$ . Note that  $\langle \mathbf{a}_{\mathcal{T},1} \rangle = 0$ , and  $\langle f \rangle$  evolves on the transport time scale.

The linear version of the theory is given by

$$\mathcal{V}_0(\tilde{f}_L) = -\partial_z \cdot \left[ \frac{-\mathbf{g}_1}{\Delta t} \langle f \rangle \right]. \quad (25)$$

Its solution is clearly

$$\tilde{f}_L = \mathbf{G}_1 \cdot \partial_z \langle f \rangle. \quad (26)$$

This recovers the previous linear theory [9, 22].

The quasilinear version is given by

$$\partial_t \langle f \rangle = -\partial_z \cdot \left[ \left\langle \frac{-\mathbf{g}_1}{\Delta t} \tilde{f}_L \right\rangle \right]. \quad (27)$$

Substituting Eq. (26), one finds the quasilinear transport equation

$$\partial_t \langle f \rangle + \partial_z \cdot \left[ -\frac{1}{2} \partial_t \langle \mathbf{G}_1 \mathbf{G}_1 \rangle \cdot \partial_z \langle f \rangle \right] = 0. \quad (28)$$

This recovers the previous quasilinear theory [7, 9].

The nonlinear transport equation can be given by

$$\partial_t \langle f \rangle + \partial_z \cdot \left[ \left( \langle \mathbf{a}_{\mathcal{T},2} \rangle - \frac{1}{2} \langle \mathbf{d}_{\mathcal{T}} \rangle \cdot \partial_z \right) \langle f \rangle \right] + \partial_z \cdot \left[ \langle \mathbf{a}_{\mathcal{T}} \rangle \tilde{f} \right] = \dots, \quad (29)$$

where the last term reduces to the quasilinear transport in the quasilinear regime according to Eq. (27) and Eq. (28).

The nonlinear diffusion tensor  $\langle \mathbf{d}_{\mathcal{T}} \rangle$  and convection vector  $\langle \mathbf{a}_{\mathcal{T},2} \rangle$  are attributed to the stochastic turbulence scattering effects in the nonlinear regime; they are important only for the stochastic motion. Comparing Eq. (26) and Eq. (28), one concludes that the quasilinear transport is due to the long-time phase coherence of the particle excursion  $\mathbf{G}_1$  and the distribution fluctuation  $\tilde{f}$ . In the nonlinear regime, this phase coherence is broken by the developed stochasticity. Therefore, one expects that the stochastic transport dominates in the nonlinear regime, and the quasilinear transport dominates in the quasilinear regime.

The symmetry of the phase space quasilinear diffusion tensor has been used to demonstrate the Onsager symmetry relation of quasilinear transport [9], as is similar to the Onsager relation of collisional transport due to the symmetry of collisional velocity space diffusion tensor [2, 4]. Noting the symmetry of the turbulent diffusion tensor in phase space, one expects that the Onsager relation still holds in the nonlinear regime, an important point for experimental data analysis.

The nonlinear evolution of the fluctuating part of distribution is given by

$$[\mathcal{V}_0 + \langle \mathbf{a}_{\mathcal{T},2} \rangle \cdot \partial_z] \tilde{f} = -\partial_z \cdot \left[ \left( \mathbf{a}_{\mathcal{T},1} \langle f \rangle - \frac{1}{2} \langle \mathbf{d}_{\mathcal{T}} \rangle \cdot \partial_z \right) \tilde{f} \right] + \dots \quad (30)$$

Note that a turbulent thermal conduction term [similar to  $\langle \mathbf{d}_{\mathcal{T}} \rangle$ ] was heuristically added to the drift kinetic equation [similar to Eq. (30)] by Kadomtsev and Pogutse [5] to solve for the saturation level of  $\tilde{f}$ , by invoking Landau's dissipation cascade argument.



For simplicity, we introduce the eikonal ansatz,  $\tilde{f} = f_{\mathbf{k}} e^{-i(\omega t - \mathbf{k} \cdot \mathbf{x})}$ . When the turbulence is saturated, Eq. (30) should balance. The real terms are from the right-hand side; the first term  $\sim \gamma_{L,\mathbf{k}} \tilde{f}$  is the linear driving term, with  $\gamma_{L,\mathbf{k}}$  the linear growth rate; and the second term  $\sim -\langle \mathbf{d}_{\mathcal{T}}^{xx} \rangle k^2 \tilde{f}$  is the nonlinear damping [23] or the nonlinear resonance broadening [1] due to the turbulent diffusion. Balancing the real terms results to the well-known formula [1, 5, 23] of turbulent diffusion coefficient

$$|\langle \mathbf{d}_{\mathcal{T}}^{xx} \rangle| \sim \frac{\gamma_{L,\mathbf{k}}}{k^2}, \quad (31)$$

which can also be obtained by invoking Prandtl's "mixing-length" argument [1, 23]. The discussions presented here and following Eq. (24) make it clear that the new turbulence scattering term collects the mysterious dissipation term in Kadomtsev-Pogutse's equation to Frieman-Chen's equation.

Balancing the imaginary terms of Eq. (30) results to the nonlinear resonance condition

$$\omega - \mathbf{k} \cdot \mathbf{v} - \mathbf{k} \cdot \left[ \langle \mathbf{a}_{\mathcal{T},2} \rangle - \frac{1}{2} \partial_z \cdot \langle \mathbf{d}_{\mathcal{T}} \rangle \right]^x \sim 0. \quad (32)$$

This indicates a nonlinear phase shift effect, which can be related to important topics widely discussed in nonlinear optics [24] and plasma physics [25]: the nonlinear frequency shift (e. g. chirping) of a wave resonantly excited with a fixed  $\mathbf{k}$ , and the nonlinear  $\mathbf{k}$  spectrum shift (e. g. self focusing) in propagation of a wave excited with a fixed  $\omega$ . This nonlinear phase shift may be related to the ponderomotive effects [9].

Although the above important points, from the linear and quasilinear to the nonlinear physics, can be successfully identified from the turbulence scattering term, it is still a complicated nonlinear term; however, it is similar to the collisional scattering term. With the two scattering terms and a possible driving source term  $\mathcal{S}$  included, the kinetic equation can be written as

$$\mathcal{V}_0(f) = \mathcal{T}(f) + \mathcal{C}(f) + \mathcal{S}. \quad (33)$$

It is useful to point out that the new method involves only integrating along the unperturbed orbit, instead of integrating along the full orbit in the conventional method. This may bring about advantages by making use of our knowledge of the well-understood unperturbed orbit, especially for the toroidal axisymmetric tokamak fusion system [22, 26–28]. It is not hard to extend this theory to the gyrokinetic version that decouples the fast gyro-motion for the low-frequency turbulence [9, 12].

In conclusion, the nonlinear turbulence scattering term has been found by using the I-transform method to decouple the turbulent motion from the unperturbed motion. A unified kinetic theory has been formulated to include both the turbulence scattering effect and the collisional scattering effect on plasma transport. The evolution of the plasma distribution function is governed by the ballistic motion of charged particles along the orbit determined by the averaged electromagnetic fields, with the effects of fluctuating part of fields described by the turbulence scattering term, from which various important physics scattered in the literature can be identified in a unified way. Particularly, the proposed turbulence scattering term reveals the collections between two widely used equations: Kadomtsev-Pogutse's equation with a mysterious dissipation term and Frieman-Chen's equation without the explicit dissipation term.

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